Statistics

Lecture 9

Feb 19-8:47 AM

Class Quiz 5

L1	L2
x	$P(x)$
1	.25
2	.2
3	.1
4	.2
5	.25

Find

1) $P(x=3)$

$= 1 - [.25 + .2 + .2 + .25] = 1 - .9 = \boxed{.1}$

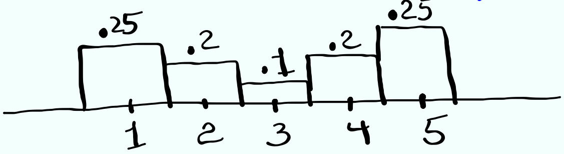
2) Find σ^2 in reduced fraction.

$\boxed{\text{1-Var Stats}}$ with L1 & L2

$\boxed{\text{VARS}}\ \boxed{\text{5: Statistics}}\ \boxed{4{:}\ \sigma x}$ $\sigma^2 = \frac{12}{5}$

$\boxed{x^2}\ \boxed{\text{Math}}\ \boxed{\text{1:}\blacktriangleright\text{Frac}}\ \boxed{\text{Enter}}$

3) Draw Prob. dist. histogram.



Oct 18-10:53 AM

Binomial Prob. dist. (SG 16)

n # of trials

p → Prob. of Success per trial $q=1-p$

x → # of Successes

0, 1, 2, 3, - - -, n

$$P(x) = {}_{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

Consider a binomial Prob. dist with

$n=20$ & $P=.6$

$q=1-P=.4$

P(exactly 14 Successes)

$$P(x=14) = {}_{20}C_{14} \cdot (.6)^{14} \cdot (.4)^{6} \approx \boxed{.124}$$

TI Command

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

$$P(x=14) = \text{binompdf}(\overset{n}{20}, \overset{p}{.6}, \overset{x}{14}) = \boxed{.124}$$

Oct 25-8:12 AM

Suppose we flip a coin 50 times.

Success is to land tails.

It is not a fair coin → P(land tail)=.7

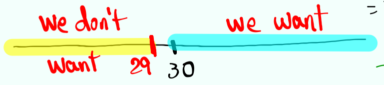
$n=50$ $P=.7$ $q=.3$

$np=50(.7)=\boxed{35}$ $npq=50(.7)(.3)=\boxed{10.5}$ $\sqrt{npq}=\sqrt{10.5} \approx \boxed{3.240}$

P(exactly 32 tails) $= P(x=32) = \text{binompdf}(50, .7, 32) = \boxed{.077}$

P(at most 40 tails) $= P(x \le 40) = \text{binomcdf}(50, .7, 40) = \boxed{.960}$

P(at least 30 tails) $= P(x \ge 30)$



$= \underset{\text{Total Prob.}}{1} - P(x \le 29)$

$= 1 - \text{binomcdf}(50, .7, 29)$

$= \boxed{.952}$

Oct 25-8:21 AM

Consider a binomial prob. dist. with $n=250$ and $P=.8$

1) $q=\boxed{.2}$

2) $np=250(.8)=\boxed{200}$

3) $npq=250(.8)(.2)=\boxed{40}$

4) $\sqrt{npq}=\sqrt{40} \approx \boxed{6.325}$

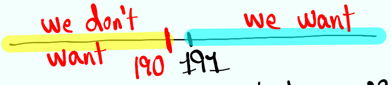
5) $P(\underbrace{\text{fewer than } 210} \text{ successes})$

$$P(x<210)=P(x\leq 209)$$
$$=\text{binomcdf}(250,.8,209)$$
$$=\boxed{.936}$$

6) $P(\underbrace{\text{more than } 190} \text{ Successes})$

$$P(x>190)=P(x\geq 191)=1-P(x\leq 190)$$

↑ Total Prob.

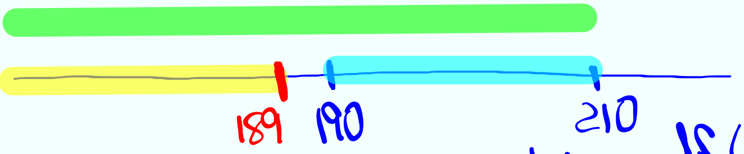


$$=1-\text{binomcdf}(250,.8,190)=\boxed{.931}$$

Oct 25-8:32 AM

7) P(# of Successes is between 190 and 210, inclusive)

$$P(190\leq x\leq 210)=P(x\leq 210)-P(x\leq 189)$$
$$=\boxed{.904}$$



$$=\text{binomcdf}(250,.8,210)-\text{binomcdf}(250,.8,189)=$$

Oct 25-8:41 AM

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard deviation $\sigma = \sqrt{\sigma^2}$

Binomial Prob. Dist.

You are taking a True/False exam with 100 questions.

You are making random guesses.

Success is to guess correctly.

$n = 100$ $p = .5$ $q = .5$

$\mu = np = 100(.5) = \boxed{50}$

$\sigma^2 = npq = 100(.5)(.5) = \boxed{25}$

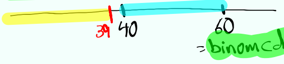
$\sigma = \sqrt{\sigma^2} = \sqrt{25} = \boxed{5}$

about 68% Range $\mu \pm \sigma = 50 \pm 5 \Rightarrow \boxed{45 \text{ to } 55}$

about 95% Range
Usual Range $\mu \pm 2\sigma = 50 \pm 2(5) \Rightarrow \boxed{40 \text{ to } 60}$

P(guess correctly between 40 to 60, inclusive)

$P(40 \le x \le 60) = P(x \le 60) - P(x \le 39)$



= binomcdf(100, .5, 60) − binomcdf(100, .5, 39)

= $\boxed{.965}$ ≈ 96.5%

Oct 25-8:47 AM

Consider a binomial Prob. dist. with

$n = 175$ & $P = .6$.

1) $q = \boxed{.4}$

2) $\mu = np = 175(.6) = \boxed{105}$

3) $\sigma^2 = npq = (175)(.6)(.4) = \boxed{42}$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{42} \approx \boxed{6.5}$

5) 95% Range $\mu \pm 2\sigma = 105 \pm 2(6.5)$

$= 105 \pm 13$

$\Rightarrow \boxed{92 \text{ to } 118}$

# of Successes ↓

6) $P(92 \le x \le 118) =$ binomcdf(175, .6, 118) − binomcdf(175, .6, 91)

Reduce by 1

= $\boxed{.963}$

SG 16 ✓

Page 4 → use exact value for p & q.

Oct 25-8:59 AM

SG 17

Geometric Prob. Dist.

It is very Similar to binomial Prob.dist. except

1) There is no n.

2) x is the number where first Success happens.

$\boxed{P(x) = p \cdot q^{x-1}}$ $x = 1, 2, 3, \ldots$

$p \to$ Prob. of Success $p + q = 1$

$q \to$ Prob. of failure $q = 1 - p$

$\mu = \frac{1}{p}$ $\sigma^2 = \frac{q}{p^2}$ $\sigma = \sqrt{\sigma^2}$

Oct 25-9:20 AM

Consider a geometric Prob. dist with $p = .5$

$q = 1 - p = \boxed{.5}$ $\mu = \frac{1}{p} = \frac{1}{.5} = \boxed{2}$

$\sigma^2 = \frac{q}{p^2} = \frac{.5}{.5^2} = \boxed{2}$ $\sigma = \sqrt{\sigma^2} = \sqrt{2} \approx 1.414 \approx \boxed{1}$

68% Range $\mu \pm \sigma = 2 \pm 1 \Rightarrow \boxed{1 \text{ to } 3}$

P(First Success happens $\underbrace{\text{on 3rd}}$ trial)

$P(x=3) = (.5)(.5)^{3-1} = (.5)(.5)^2 = \boxed{.125}$

$P(x) = p \cdot q^{x-1}$

TI Command $P(x=3) = \text{geometPdf}(.5, 3) = \boxed{.125}$

Oct 25-9:25 AM

P(first success happens before the 3rd trial)

$P(x < 3) = P(x \leq 2) = \text{geometcdf}(.5, 2)$

$= \boxed{.75}$

P(land tails) = .6 on a loaded coin.

$q = .4$ $\mu = \frac{1}{p} = \frac{1}{.6} = 1.\overline{6} \approx 2$

$\sigma^2 = \frac{q}{p^2} = \frac{.4}{.6^2} = \boxed{1.\overline{1}}$ $\sigma = \sqrt{\sigma^2} = \sqrt{1.111} \approx \boxed{1}$

Usual Range $\mu \pm 2\sigma = 2 \pm 2(1) \Rightarrow 0$ to 4

P(First tail happens on 3rd or 5th trial)

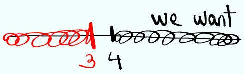
$P(x = 3 \text{ OR } x = 5)$

$= \text{geometpdf}(.6, 3) + \text{geometpdf}(.6, 5)$

$= \boxed{.111}$

P(first tail happens after 3rd trial)

$P(x > 3) = P(x \geq 4) = 1 - P(x \leq 3)$



$= 1 - \text{geometcdf}(.6, 3)$

$= \boxed{.064}$

Oct 25-9:31 AM

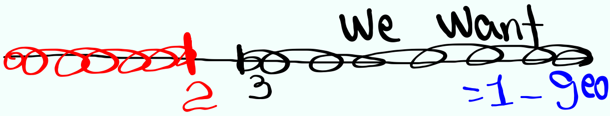
Prob. that LeBron James makes FT is .8.

P(He makes First FT on 2nd attempt)

$P(x = 2) = \text{geometpdf}(.8, 2) = \boxed{.16}$

P(He makes first FF after 2nd attempt)

$P(x > 2) = P(x \geq 3) = 1 - P(x \leq 2)$



$= 1 - \text{geometcdf}(.8, 2) = \boxed{.04}$

Oct 25-9:41 AM

SG 17

Poisson Prob. dist μ or λ

use this when $\boxed{\text{average \# of Successes}}$ per fixed interval is given.

$$P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu} \qquad x = 0, 1, 2, 3, \ldots$$

$$e \approx 2.718$$

$$\sigma^2 = \mu \qquad \sigma = \sqrt{\sigma^2}$$

Oct 25-9:47 AM

Consider a poisson Prob. dist. with $\mu = 9$ on a fixed interval.

$$\sigma^2 = \mu = 9 \qquad \sigma = \sqrt{\sigma^2} = \sqrt{9} = 3$$

68% Range $\mu \pm \sigma = 9 \pm 3 \rightarrow \boxed{6 \text{ to } 12}$

$P(x = 10)$ = Poisson pdf$(9, 10)$ = $\boxed{.119}$

$P(x \leq 12)$ = Poisson cdf$(9, 12)$ = $\boxed{.876}$ ≈88%

Oct 25-9:51 AM

John works at a support center.

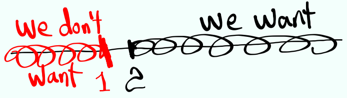
He gets in $\underbrace{\text{average } 4}_{\mu=4}$ Calls $\underbrace{\text{per hr.}}_{\text{Fixed interval}}$

P(He gets 6 Calls in one hr)

$P(x=6) = \text{Poissonpdf}(4,6) = \boxed{.104}$

P(He gets $\underbrace{\text{at least 2}}$ Calls in one hr).

$P(x \geq 2) = 1 - P(x \leq 1) = 1 - \text{Poissoncdf}(4,1)$



$= \boxed{.908}$

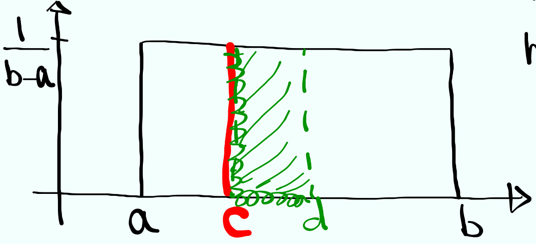
SG 17 ✓

Oct 25-9:57 AM

SG 18

Working with Continuous Random Variable & Prob. dist.

$\underbrace{\text{Uniform Prob. Dist.}}$ $\underbrace{\text{For all values from a to b:}}$ → graph is rectangula from a to b with height of $\frac{1}{b-a}$.



Line

(has Zero Area)

1) $P(x=c)=0$

2) Total Area $=1$

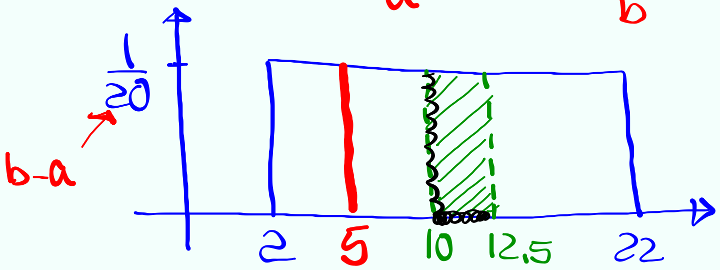
$P(c<x<d)$

$=(d-c)\cdot\frac{1}{b-a}$

Oct 25-10:14 AM

Consider a $\underbrace{\text{uniform Prob. dist}}$ for all Values from $\underset{\uparrow \atop a}{2}$ to $\underset{\uparrow \atop b}{22}$.

↳ Rectangular graph



1) $P(x=5) = 0$

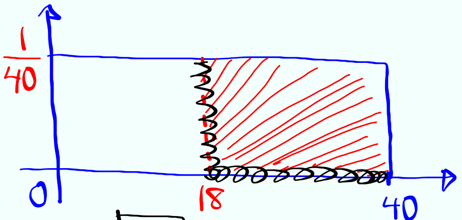
2) $P(10<x<12.5)$

$=(12.5-10)\cdot\frac{1}{20}$

$= \frac{2.5}{20} = \boxed{\frac{1}{8}}$

Oct 25-10:20 AM

Consider a Uniform Prob. dist. for all Values from 0 to 40.

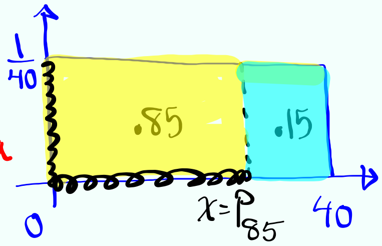


$P(x>18)=(40-18)\cdot\frac{1}{40} = \frac{22}{40} = \boxed{\frac{11}{20}}$

Find $x = P_{85}$

85% below
Left Area
.85

15% above
Right area
.15

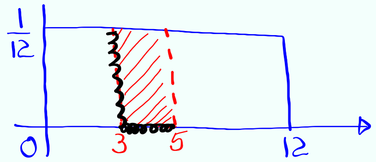


$(x-0)\cdot\frac{1}{40} = .85$ $\quad x = 40(.85) = \boxed{34}$

Oct 25-10:25 AM

City bus comes around every 12 mins.

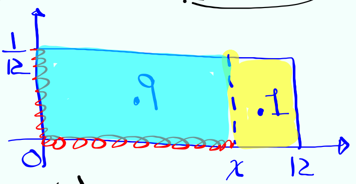
Let x be the wait time.



P(Your wait time is between 3 to 5 mins)

$$P(3<x<5) = (5-3)\cdot\frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$

Find the wait time that separates the top 10% from the rest. Round to whole min.

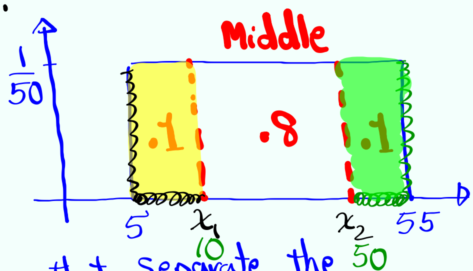


$(x-0)\cdot\frac{1}{12} = .9$ $\quad x = 12(.9)$ $\quad x = 10.8 \approx \boxed{11}$

Oct 25-10:32 AM

Use Uniform Prob. dist. for all values from 5 to 55.

1) Draw & label.



2) Find two values that separate the middle 80% from the rest.

$(x_1-5)\cdot\frac{1}{50} = .1$ $\quad x_1-5 = 50(.1)$

$x_1-5 = 5$ $\quad \boxed{x_1=10}$

$(55-x_2)\cdot\frac{1}{50} = .1$ $\quad 55-x_2 = 50(.1)$

$55-x_2 = 5$

$\boxed{x_2=50}$

Oct 25-10:39 AM

Standard Normal Prob. dist.

1) use Z, $P(Z=c)=0$

2) Data dist. is symmetric and has a bell-shape curve with total Area $=1$.

3) Mean = Mode = Median

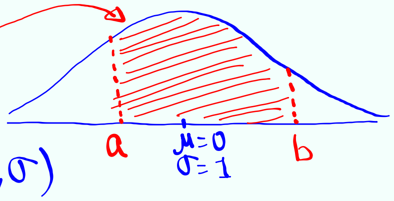
4) $\mu=0$ & $\sigma=1$

$\boxed{P(a<Z<b)}$ is the Corresponding area within the bell-Curve.

How to find it:

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

normalcdf(L, U, μ, σ)



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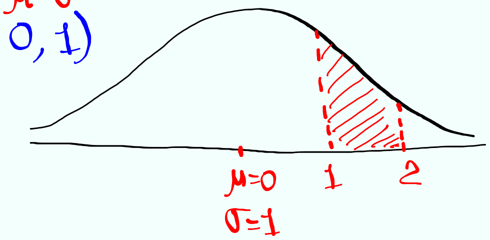
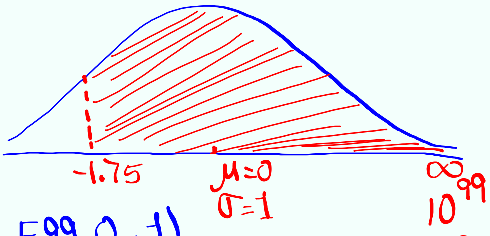
$P(1<Z<2)$

$=\text{normalcdf}(\overset{L}{1},\overset{U}{2},\overset{\mu}{0},\overset{\sigma}{1})$

$=\boxed{.136}$



$P(Z>-1.75)$



$E\,99$

$\boxed{\text{2nd}}$ $\boxed{,}$

$=\text{normalcdf}(-1.75, E99, 0, 1)$

$(-)$

$=\boxed{.960}$

Oct 25-10:51 AM

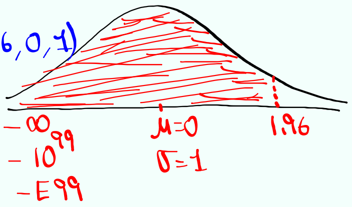
$P(Z < 1.96)$

$= \text{normalcdf}(-E99, 1.96, 0, 1)$

(-)

$\boxed{2nd}\ \boxed{9}$

$\approx \boxed{.975}$



Find $Z = P_{98}$

98% below

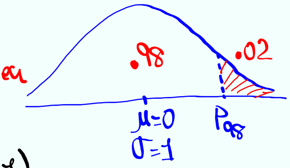
Left Area

.98

2% above

Right Area

.02



$Z = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

left Area

μ

σ

Oct 25-10:57 AM